AE-806

M.A./M.Sc. (Final)
Term End Examination, 2016-17

MATHEMATICS

Compulsory

Paper - II

Partial Differential Equations,
Mechanics and Gravitation

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

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1. (*a*) (*i*) Find $L^{-1}\left\{\dfrac{p+2}{p^2-4p+13}\right\}$

(*ii*) Solve the differential equation

$$\frac{\partial^2 y}{\partial x^2}-\frac{\partial^2 y}{\partial t^2}=xt, \text{ where}$$

$$y=0=\frac{\partial y}{\partial t} \text{ at } t = 0 \text{ and } y(0,\ t) = 0$$

by using Laplace Transform.

(*b*) Find the Fourier transform of $f(x)$ defined by

$$f(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$$ and hence evaluate

(*i*) $$\int_{-\infty}^{\infty}\frac{\sin pa\cos px}{p}\,dp$$

(*ii*) $$\int_{0}^{\infty}\frac{\sin p}{p}\,dp$$

2. (*a*) (*i*) Solve the Differential Equation

$$(t+y+z)\frac{\partial t}{\partial x}+(t+x+z)\frac{\partial t}{dy}+(t+x+y)\frac{\partial t}{dz}=x+y+z$$

(*ii*) Classify the following equation :

$$(1-x^2)\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+(1-y^2)\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}+3x^2y\frac{\partial z}{\partial y}-2=0$$

(*b*) Show that the Green's function $G(r, r')$ has the symmetric property.

3. (*a*) Define the Harmonic function. If a harmonic function vanishes everywhere on the boundary, then show that it is identically zero everywhere.

(*b*) Derive the one-dimensional wave equation.

4. (*a*) Let u be a harmonic function in the interior of a rectangle $0 \le x \le a$, $0 \le y \le b$ in xy plane satisfying Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with $u(0, y) = 0$, $u(a, y) = 0$,
$u(x, b) = 0$, $u(x, 0) = f(x)$

Determine u for above problem, by using the method of separation of variables.

(*b*) Find the fundamental solution of Heat equation.

5. (*a*) Derive Routh's Equations.

(*b*) Give the definition of cyclic coordinates and show that the generalised momentum conjugate to a cyclic coordinate is conserved.

6. (*a*) Show that Poisson's Brackets are invariant under canonical transformation.

(*b*) Derive Hamilton-Jacobi Equation.

7. (*a*) State and prove Donkin's Theorem.

(*b*) Define the following :

(*i*) Generalised coordinates

(*ii*) Poisson Brackets

(*iii*) Hamiltonian

8. (*a*) Solve the Brachistochrone problem.

(*b*) Show that the transformation defined by

$$q = \sqrt{[2P]}\sin Q$$

$$p = \sqrt{[2P]}\cos Q$$

is canonical.

9. (*a*) Find the attraction of thin uniform spherical shell at an external and internal point.

(*b*) Show that the potential of a uniform circular disc, of mass M and radius a, at a point in its plane distant c from its centre, is

$$\frac{4\gamma M}{\pi a^2}\int_0^{\frac{\pi}{2}}\sqrt{a^2-c^2\sin^2\theta}\ d\theta \quad \text{or}$$

$$\frac{4\gamma M}{\pi a^2}\int_0^{\sin^{-1}\frac{a}{c}}\sqrt{a^2-c^2\sin^2\theta}\ d\theta$$

according as c is less or greater than a.

10. (*a*) Derive the Poisson Equation.

(*b*) Find the distribution of matter which will produce the following potentials :

$V = 1$ within the ellipsoid

$$\frac{x^2}{\mu^2a^2}+\frac{y^2}{\mu^2b^2}+\frac{z^2}{\mu^2c^2}=1;\ (\mu<1)$$

$$V=\frac{1}{1-\mu^2}\left[1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}\right]$$

between the above ellipsoid and

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1;$$

$V = 0$ outside the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$