AE-799

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Compulsory

Paper - I

Advanced Abstract Algebra

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

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1. (*a*) Show that any subgroup of a solvable group is solvable.

(*b*) Let G be a nilpotent group. Then show that every subgroup of G and every homomorphic image of G are nilpotent.

2. (*a*) Define Normal Series, Subnormal Series and Composition Series with example .

(*b*) State and prove Jordan-Holder theorem for finite group.

3. (*a*) Define Permutation group. Prove that every group is isomorphic to a permutation group.

(*b*) Let G be a group and let G' be the derived group of G. Then show that—

(*i*) $G' \Delta G$;

(*ii*) G / G' is abelian ;

(*iii*) if $H \Delta G$, then $G'\ H$ is abelian if and only if $G' \subset H$.

4. (*a*) State and prove Third isomorphism theorem.

(*b*) State and prove Correspondence theorem.

5. (*a*) State and prove fundamental theorem of R-homomorphism.

(*b*) State and prove Schur's lemma.

6. (*a*) Define free modules. Let M be a finitely generated free module over a commutative ring R. Then show that all bases of M have the same number of elements.

(*b*) State and prove Rank-nullity theorem.

7. (*a*) Prove that $\mathrm{Hom}_{\mathrm{F}}(V, V) \cong F_n$ as algebras over F, when $\dim V = n$.

(*b*) Find the rank of the linear mapping

$$\phi : R^4 \to R^3$$

where

$\phi\ (a,\ b,\ c,\ d) = (a + 2b - c + d,\ -3a + b + 2c - d,\ -3a + 8b + c + d)$.

8. (*a*) Show that $x^3 + 3x + 2 \in Z/(7)[x]$ is irreducible over the field $Z/(7)$.

(*b*) Show that an element $a \in K$ is algebraic over F if and only if $[F(a) : F]$ is finite.

9. (*a*) Define algebraically closed fields with example. Let F be a field. Then show that there exists an algebraically closed field K containing F as a subfield.

(*b*) Show that the prime field of a field F is either isomorphic to Q or to $Z/(p)$, p is prime.

10. (*a*) State and prove Hilbert Basis Theorem.

(*b*) State and prove Fundamental Theorem of Galois theory.