

AE-801

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Paper - III

Topology

Time : Three Hours] [Maximum Marks : 100
[Minimum Pass Marks : 36

Note : Answer any **five** questions. All questions carry equal marks.

1. (a) Let B and B' be bases for topology T and T' respectively on X . Then prove that the following statements are equivalent :
 - (i) T' is finer than T .
 - (ii) For each $x \in X$ and each basis element $\beta \in B$ containing x , there is a basis element $\beta' \in B'$ such that $x \in \beta' \subset \beta$.

(2)

- (b) If B is a basis for topology of X , then prove that the collection $B_y = \{\beta \cap y : \beta \in B\}$ is a basis for the subspace topology on y .
2. (a) If A is a subset of the topological space X and let A' be the set of all limit points of A , then prove that $\bar{A} = A \cup A'$.
- (b) State and prove pasting lemma.
3. (a) Prove that every second countable space is first countable but converse is not true in general.
- (b) Prove that image of a connected space under a continuous map is connected.
4. (a) If A is a connected subspace of X and $A \subset B \subset \bar{A}$, then prove that B is also connected.
- (b) Prove that a finite Cartesian product of connected spaces is connected.
5. (a) Prove that a relation on a topological space defined by " $x \sim y$ " \rightarrow "If there is a path from x to y " is an equivalence relation.
- (b) Prove that a space X is locally connected if and only if for every open set U of X each component of U is open in X .

(3)

6. (a) Prove that every closed subspace of a compact space is compact.
(b) Prove that every compact space is limit point compact but its converse is not true in general.
 7. (a) Prove that every compact subspace of a Hausdorff space is closed.
(b) Prove that image of a compact space under a continuous map is compact.
 8. (a) Show that every compact Hausdorff space is normal.
(b) Prove that product of regular space is regular.
 9. (a) Show that every regular space with countable basis is normal.
(b) If X is a Hausdorff space, then show that a net in X converges to at most one point.
 10. State and prove Urysohn's metrization theorem.
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