

# AE-809

M.A./M.Sc. (Final)  
Term End Examination, 2016-17

# MATHEMATICS

Optional

## Paper - V

## Difference Equations

*Time : Three Hours]                    [Maximum Marks : 100*  
*[Minimum Pass Marks : 36*

**Note** : Answer any **five** questions. All questions carry equal marks.

1. (a) Solve the difference equation :

$$(\Delta^2 - 3\Delta + 2)y_k = 0$$

- (b) Define the difference equation with example.

- 2. (a)** Solve the difference equation :

$$y_n = (A_n + B) \cdot 3^n$$

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(b) Solve the following equation :

$$u_{x+1} - e^{2x} u_x = 0$$

3. (a) Explain the Green's function.

(b) Solve the following equation :

$$y_{x+3} - 2y_{x+2} - y_{x+1} + 2y_x = 0$$

4. (a) If  $y_k$  satisfies the difference equation

$$y_{k+1} - \lambda y_k + y_{k-1} = 0, \quad k = 1, 2, 3$$

and the end conditions  $y_0 = y_u = 0$ ,  
determine  $\lambda$  for which a non-trivial  
solution exists.

(b) Solve the difference equation :

$$y_{k+1} - y_k + ky_{k+1} \cdot y_k = 0, \quad y_1 = 2$$

5. Evaluate the determinant given below  
( $n^{\text{th}}$  order) by forming the difference equation :

$$A_n = \begin{vmatrix} 2\cos\theta & 1 & 0 & 0 & \text{---} & 0 \\ 1 & 2\cos\theta & 1 & 0 & \text{---} & 0 \\ 0 & 1 & 2\cos\theta & 1 & \text{---} & 0 \\ - & - & - & - & \text{---} & - \\ 0 & - & 0 & 1 & \text{---} & 2\cos\theta \end{vmatrix}$$

6. (a) Write down the Lipschitz case.

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- (b) The conditions  $1 + C_1 + C_2 > 0$ ,  $1 - C_1 + C_2 > 0$ ,  $1 - a_2 > 0$  are necessary and sufficient for both roots of  $m^2 + C_1m + C_2 = 0$  to be less than one in absolute value. Prove.

7. (a) Find the nature of stability of the following linear system :

$$\frac{dx}{dt} = 2x - 2y + 10$$

$$\frac{dy}{dt} = 11x - 8y + 49$$

- (b) Solve the difference equation :

$$u(x+2) + u(x+1) + u(x) = x^2 + x + 1$$

8. (a) Consider non-linear system :

$$\frac{dx}{dt} = -2x + 3y + xy$$

$$\frac{dy}{dt} = -x + y - 2xy^2$$

solve by variable separation method.

- (b) Find the Lipschitz constant :

$$\frac{\partial g}{\partial u} = t^2u + u^2, |u - 1| \leq 2$$

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9. (a) Solve the boundary value problem

$y'' + y = -x$ ,  $(0 < x < 1)$ ,  $y(0) = y(1) = 0$   
by Rayleigh-Ritz method.

(b) Solve the boundary value problem  
defined by

$y'' - x = 0$ ,  $y(0) = 0$ ,  $y'(1) = \frac{-1}{2}$  by  
Rayleigh-Ritz method.

10. (a) Define the following with example :

(i) Linear system stability

(ii) Non-linear system stability

(iii) Disconjugacy

(iv) The Riccati equations

(b) Solve the following :

$$u_{x+1} - u_x + 2v_{x+1} = 0$$

$$v_{x+1} - v_x - 2u_x = 0$$