AE-804

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Optional

Paper - III

Differential Geometry of Manifolds

Time : Three Hours] [*Maximum Marks* : 100
[*Minimum Pass Marks* : 36

Note : Answer any **five** questions. All questions carry equal marks.

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1. (*a*) Give the derivation property of Tangent spaces

$$X_a(f \,.\, g) = X_a(f)\,.\,g(a) + f(a)\,.\,X_a(g)$$

(*b*) Show that $D(1) = 2D(1)$.

2. (*a*) Show that $X_x = (T_x u)^{-1} \,.\, T_x u \,.\, X_x$

(*b*) Let $\phi : G \to H$ be a smooth homomorphism of Lie groups. Then show that $\phi' = T_e\phi : g = T_eG \to h = T_eH$ is a Lie algebra homomorphism.

3. (*a*) Prove that $X^T = k_M \circ TX$ for vector bundle homomorphism.

(*b*) Show that for exterior derivative $d^2 = d \circ d = 0$ where $d : \Omega^k(M) \to \Omega^{k+1}(M)$.

4. (*a*) Show that the tangent bundle of the associated bundle $P[s, l]$ is given by

$T(P[s, l]) = TP[Ts, Tl]$

(*b*) For the pullback of a vector bundle along $f : N \to M$, then show that $p(f^*E) = f^*p(E)$.

5. Let (M, g) be a Riemann manifold with sectional curvature $k \geq k_0 > 0$, then show that for any geodesic c in M the distance between two conjugate points along c is $\leq \frac{\pi}{\sqrt{k_0}}$.

6. Let $p : N \to M$ be a surjective submersion (a fibered manifold) which is proper, so that $p^{-1}(k)$ is compact in N for each compact $k \subset M$ and let M be connected. Then show that (N, p, M) is a fibre bundle.

7. Show that the circle $S' \subset ₵$ is a Lie group under complex multiplication and the map

$$z = e^{i\theta} \rightarrow \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & I_{n-2} \end{bmatrix}$$

is a Lie group homomorphism into $SO(n)$.

8. (*a*) Show that the range of the zero section of a vector bundle $E \rightarrow M$ is a submanifold of E.

(*b*) For any $X \in \chi(M)$ and any $f \in \Omega^{o}(M)$, then show that $L_X df = dL_x f$

9. (*a*) Show that the tangent bundle is a vector bundle.

(*b*) Show that the tangent bundle of a Lie group is trivial $TG \cong G \times g$.

10. (*a*) Show that for each positive integer n the space R^n is a differential manifold.

(*b*) Show that S' embedded as $S' \times \{ 1 \}$ in the torus $S' \times S'$ is a closed subgroup.
